EW precision observables

- Schwartz 31.1, 31.2
- Barbieri's lectures 0706.0684
- EWSB after LEP. hep-ph/0405040

We have seen how radiative corrections can dominate dynamics, required to interpret LEP data even at the qualitative level. Given the precision of the measurements, also perturbatively small effects are important. There is a class of loop corrections, known as 'oblique' corrections, that are particularly important in order to interpret EW data.

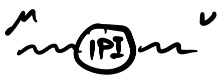
We've seen how a small set of parameters, say G_F, α_{em}, m_Z, fix all the rest of masses and couplings of the EW sector. However, this has been derived at tree level. What happens at loop level?

- QED renormalization

Let us remind what happens with loop diagrams for the photon propagator.

$$i D^{\mu\nu} = \frac{-i\eta^{\mu\nu}}{q^2 + i\varepsilon} \quad :$$ 

This gets contributions from 1PI diagrams



$$\equiv i\Pi^{\mu\nu}(q)$$

The Ward identity implies

$$q_\mu \Pi^{\mu\nu} = q_\nu \Pi^{\mu\nu} = 0$$

So we write

$$\Pi^{\mu\nu}(q) = (q^2\eta^{\mu\nu} - q^\mu q^\nu)\,\Pi(q^2)$$

↳ regular at $q^2 \to 0$.

Using this,

By summing the series,

$$iD^{\mu\nu} = \frac{-i\eta^{\mu\nu}}{q^2 + i\varepsilon} \longrightarrow iD'^{\mu\nu} = \frac{-i\eta^{\mu\nu}}{q^2(1-\Pi(q^2)) + i\varepsilon}$$

The photon propagator allways comes with factors of e.

$$ie^2 D'^{\mu\nu} = \frac{-i\eta^{\mu\nu} e_0^2}{q^2(1-\Pi(q^2))}$$

so effectivelly, the charge has to be replaced by the "running coupling"

$$e^2 \longrightarrow e^2(q^2) = \frac{e_0^2}{1-\Pi(q^2)}$$

This is fixed by the condition that at $q^2 \to 0$, one gets the measured e.m. coupling in atomic experiments

$$e^2(q^2=0) \equiv e^2 = \frac{e_0^2}{1-\Pi(0)} \quad \rightarrow \quad Z_\gamma = \frac{1}{1-\Pi(0)}$$

so it fixes the wavefunction renormalization factor.

Therefore, the propagator, in terms of the physical charge, is

$$\frac{-i\eta^{\mu\nu} e_0^2}{q^2(1-\Pi(q^2))} \simeq \frac{-i\eta^{\mu\nu} e^2}{q^2\left(1-(\Pi(q^2)-\Pi(0))\right)} \equiv \frac{-i\eta^{\mu\nu} e^2(q^2)}{q^2+i\varepsilon}$$

The quantity $\Pi(q^2)-\Pi(0)$ is finite and given by (see e.g. Peskin Eq 7.91)

$$\Pi(q^2)-\Pi(0) = \frac{2\alpha}{\pi}\int_0^1 dx\, x(1-x)\log\frac{m^2}{m^2-x(1-x)q^2}$$

For $-q^2 \gg m^2$, one gets

$$\Pi(q^2)-\Pi(0) \simeq \frac{\alpha}{3\pi}\log\frac{-q^2}{m^2}$$

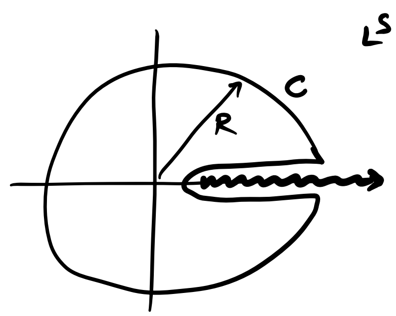
so

$$\alpha(q^2) = \frac{\alpha}{1-\frac{\alpha}{3\pi}\log\frac{-q^2}{m^2}}$$

• At the Z-pole, we want to use the e.m. coupling at that scale, $\alpha_{em}(m_Z)$. This involves evaluating $\Pi(q^2 = m_Z^2)$.

We can use dispersion relations.

Take an analytic function $F(s)$, up to a branch cut above $s > s_0$.



It satisfies

$$F(s) = \frac{1}{2\pi i} \oint \frac{ds' \, F(s')}{s'-s}$$

Since $F(s^*) = F^*(s)$ (our fn is real analytic)

the cut gives

$$\lim_{\varepsilon \to 0} F(s+i\varepsilon) - F(s-i\varepsilon) = 2i \, \mathrm{Im}\, F(s)$$

so for $R \to \infty$,

$$F(s) = \frac{1}{\pi} \int_{4m^2}^{\infty} ds' \, \frac{\mathrm{Im}\, F(s')}{s'-s} + C_\infty$$

If $F(s)$ falls off sufficiently fast for $s \to \infty$, then $C_\infty \to 0$.

In general, this might not be the case, and the "subtracted" dispersion relation is needed.

$$F(s) - F(0) = \frac{1}{\pi} \int_{4m^2}^{\infty} \mathrm{Im}\, F(s') \left(\frac{1}{s'-s} - \frac{1}{s'} \right)$$

$$= \frac{s}{\pi} \int_{4m^2}^{\infty} \frac{ds'}{s'} \, \frac{\mathrm{Im}\, F(s')}{s'-s}$$

This is exactly the structure we have for the photon self-energy. While $\Pi(q^2)$ diverges, $\Pi(q^2) - \Pi(0)$ is finite and it obeys a dispersion relation that allows us to relate it with its imaginary part,

$$\Pi(q^2) - \Pi(0) = \frac{q^2}{\pi}\int_{4m^2}^{\infty} \frac{ds}{s}\,\frac{\mathrm{Im}\,\Pi(s)}{s-q^2}$$

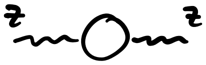
Using now the optical theorem, we can relate the $\mathrm{Im}\,\Pi$ to the total cross section $\sigma(e^+e^- \to \gamma^* \to \text{anything})$.

At high q^2, this is calculable in perturbation theory.
At low q^2, it is more problematic due to hadronization, but then one can rely on experimental data.

- EW precision observables

The gauge boson self-energies encode the corrections that will affect all processes, similarly to running α in QED.
Now, the self-energy affects relations between input parameters.

The Z-boson self-energy,



$$= i\,\Pi_{ZZ}\,\eta^{\mu\nu} + i\,\Pi_{ZZ}^{pp}\,\underbrace{p^\mu p^\nu}$$

term small for coupling to light fermions.

After resummation,

$$i\,G_Z^{\mu\nu} \simeq \frac{-i\,\eta^{\mu\nu}}{p^2 - m_Z^2 - \Pi_{ZZ}(p^2)}$$

$$\rightarrow \quad m^2_{Z,\text{pole}} = m_Z^2 + \text{Re}\left[\Pi_{ZZ}(m_Z^2)\right]$$

$$m^2_{W,\text{pole}} = m_W^2 + \text{Re}\left[\Pi_{ZZ}(m_Z^2)\right]$$

Note that Ward id. forces

$$\Pi^{\mu\nu}_{\gamma\gamma} = (q^2\eta^{\mu\nu} - q^\mu q^\nu)\,\Pi_{\gamma\gamma}(q^2)$$

so $\Pi^{\mu\nu}_{\gamma\gamma}(0) = 0$ & photon is massless. Instead, the Higgs vev gives a contribution to Π_{ZZ} & Π_{WW} at zero momentum.

We derived that G_F is given by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8 m_W^2}$$

by computing the coefficient of the weak current. This is at tree level. Taking into account loop corrections,

$$\frac{G_F}{\sqrt{2}} = - \frac{g^2}{8} \frac{1}{p^2 - m_W^2 - \Pi_{WW}(p^2)} \Big|_{p^2 \to 0}$$

$$= \frac{g}{8 m_W^2} \left(1 - \frac{\Pi_{WW}(0)}{m_W^2} + \ldots \right)$$

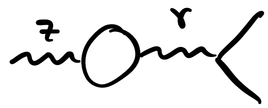
The coupling of leptons to the Z boson, which is measured very precisely, does also receive contributions at one loop.

The tree level Lagrangian

$$\mathcal{L}_{\gamma Z} = -\frac{e}{sc} Z_\mu \left[\left(\tfrac{1}{2} - s^2\right) \bar{e}_L \gamma^\mu e_L - s^2 \bar{e}_R \gamma^\mu e_R \right]$$

$$- e A_\mu \left(\bar{e}_L \gamma^\mu e_L + \bar{e}_R \gamma^\mu e_R \right)$$

has one-loop corrections



which contribute by a factor of $\Pi_{\gamma Z}/p^2$, with $p^2 = m_Z^2$, the physical Z-boson mass.

This leads to

$$\mathcal{L}_Z = -\frac{e}{sc} Z_\mu \left[\left(\tfrac{1}{2} - s^2\right) \bar{e}_L \gamma^\mu e_L - s^2 \bar{e}_R \gamma^\mu e_R \right]$$

$$- e \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2} Z_\mu \left[\bar{e}_L \gamma^\mu e_L - \bar{e}_R \gamma^\mu e_R \right]$$

$$= \frac{e}{sc} Z_\mu \left[\left(\tfrac{1}{2} - s^2_{eff}\right) \bar{e}_L \gamma^\mu e_L - s^2_{eff} \bar{e}_R \gamma^\mu e_R \right]$$

with

$$s^2_{eff} \equiv s^2 - sc \frac{\Pi_{\gamma Z}(m_Z^2)}{m_Z^2}$$

So the correction is captured by a redefinition of the mixing angle.

This is because what $\Pi_{\gamma Z}$ does is to give an off-diagonal component to the photon - Z mass matrix. Diagonalization is thus performed by redefining the mixing angle.

The 3 input parameters, α_{em}, G_F, m_Z discussed at tree level, out of which the rest can be derived, are substituted by three input observables,

$$\hat{e}(m_Z^2) = e^2\left(1 + \frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right)$$

$$\hat{G}_F = \sqrt{2}\,\frac{e^2}{8 s^2 c^2 m_Z^2}\left(1 - \frac{\Pi_{WW}(0)}{m_W^2}\right)$$

$$\hat{m}_Z = m_Z^2 + \Pi_{ZZ}(m_Z^2)$$

If we invert these relations to write the parameters in terms of observables,

$$e^2 = \hat{e}^2\left(1 - \frac{\Pi_{\gamma\gamma}(m_Z^2)}{m_Z^2}\right)$$

$$m_Z^2 = \hat{m}_Z^2 - \Pi_{ZZ}(m_Z^2)$$

$$s^2 c^2 = \sqrt{2}\,\frac{\hat{e}^2}{8\hat{G}_F\,\hat{m}_Z^2}\left(1 - \frac{\Pi_{\gamma\gamma}(m_Z^2)}{\hat{m}_Z^2} - \frac{\Pi_{ZZ}(m_Z^2)}{\hat{m}_Z^2} - \frac{\Pi_{WW}(0)}{\hat{m}_W^2}\right)$$

we can also write the mixing angle s^2 in terms of

$$\hat{s}^2 \cdot (1-\hat{s}^2) \equiv \frac{\pi \, \hat{\alpha}_e(m_Z^2)}{\sqrt{2}\, \hat{G}_F \, \hat{m}_Z^2}$$

so $\hat{s}$ is the tree-level expression of the mixing angle in terms of (α, G_F, m_Z), but using the observables instead.

The observed W-boson mass is

$$m_{W,pole}^2 = c^2 m_Z^2 + \Pi_{WW}(m_Z^2)$$

$$= \hat{c}^2 \hat{m}_Z \left(1 - \frac{\hat{s}^2}{\hat{c}^2 - \hat{s}^2} \Pi_R - \frac{\Pi_{ZZ}}{m_Z^2} + \frac{\Pi_{WW}}{m_Z^2} \right)$$

$$\downarrow$$

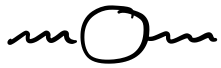
$$\equiv - \frac{\Pi_{\gamma\gamma}}{\hat{m}_Z^2} + \frac{\Pi_{ZZ}}{\hat{m}_Z^2} - \frac{\Pi_{WW}(0)}{\hat{m}_W^2}$$

similarly for the effective mixing angle

$$s_{eff}^2 \equiv \hat{s}^2 + \frac{\hat{s}^2 \hat{c}^2}{\hat{c}^2 - \hat{s}^2} \Pi_R - \hat{s}\hat{c} \, \frac{\Pi_{\gamma Z}(m_Z^2)}{\hat{m}_Z^2}$$

• The ρ parameter

We can compute the selfenergies

 $= i\,\Pi^{\mu\nu}$

in order to get an explicit expression for $\Pi_{VV'}$, see e.g. Schwartz 31.1.

The terms proportional to fermion masses have nothing to do with gauge symmetry, and are a probe of EWSB.

One can work then in a gauge-less theory.

The Higgs potential is a function of

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} h_3 + i h_4 \\ h_1 + i h_2 \end{pmatrix}$$

$$, \quad H^\dagger H = \frac{1}{2}\left(h_1^2 + h_2^2 + h_3^2 + h_4^2 \right)$$

so it is invariant under a larger global $SO(4)$ symmetry, with (h_1, h_2, h_3, h_4)

being in the fundamental.

$SO(4) \simeq SU(2) \times SU(2)$, of which the gauged $SU(2) \times U(1)$ is a subgroup.

By defining

$$\mathcal{H} = (i\sigma_2 H^*, H)$$

the action of $SU(2)_L \times SU(2)_R$ can be identified as

$$\mathcal{H} \longrightarrow e^{i\omega_L^i \frac{\sigma^i}{2}} \mathcal{H} \, e^{-i\omega_R^i \frac{\sigma^i}{2}}$$

the global $SU(2)_L$ actually coincides with the gauged $SU(2)$. Therefore, the gauging respects the full $SU(2)_L \times SU(2)_R$, with W_L^i being a triplet of $SU(2)_L$.

Gauging the hypercharge breaks the global symmetry.

Since the Higgs vev $SO(4) \rightarrow SO(3)$, or $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$, this explains the degeneracy of masses of EW bosons for $g' \to 0$.

In the gauge-less theory,

$$\mathcal{L} = i\bar{\Psi}\not{D}\Psi + |D_\mu H|^2 - V(H) - y_t H^c \bar{Q}_L t_R + h.c.$$

the relevant interactions is with the third generation of quarks, due to top Yukawa.

Writing

$$H(x) = e^{i\pi^i(x)\frac{\sigma^i}{2}} \begin{pmatrix} 0 \\ \frac{v+h(x)}{\sqrt{2}} \end{pmatrix}$$

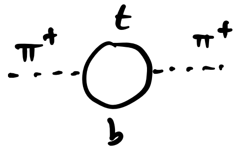
one gets, for the goldstone kinetic term,

$$\mathcal{L}_{kin} = Z_2^{(+)} \left|\partial_\mu \pi^+ - g\frac{v}{\sqrt{2}} W^+\right|^2 + \frac{Z_2^0}{2}\left(\partial_\mu \pi^0 - \frac{gv}{2c_w} Z_\mu\right)^2$$

which allows to define the rho parameter in terms of the goldstone wave func. renorm.

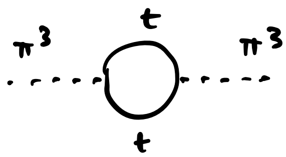
$$\rho = \frac{m_w^2}{m_Z^2 c_w^2} = \frac{Z_2^{(+)}}{Z_2^0}$$

This allows to compute the rho param. as the ratio of



$$\simeq \frac{3}{16\pi^2 v^2} m_t^2 \left(\Delta + \frac{1}{2} - \ln \frac{m_t^2}{\mu^2} \right) p^2 + \dots$$

$m_b,\ p^4$



$$\simeq \frac{3}{16\pi^2 v^2} m_t^2 \left(\Delta - \ln \frac{m_t^2}{\mu^2} \right) p^2 + \dots$$

with $\Delta = \frac{1}{\epsilon} - \gamma_E + \ln 4\pi$ being the div. in dim reg.

Therefore,

$$\rho = \frac{Z_2^{(+)}}{Z_2^0} \simeq \frac{1 + \frac{3}{16\pi^2} y_t^2 \left(\Delta + \frac{1}{2} - \ln \frac{m_t^2}{\mu^2} \right)}{1 + \frac{3}{16\pi^2} y_t^2 \left(\Delta - \ln \frac{m_t^2}{\mu^2} \right)}$$

$$\simeq 1 + \frac{3}{32\pi^2} y_t^2$$

$$= 1 + \frac{3}{8} \frac{G_F}{\sqrt{2}} \frac{m_t^2}{\pi}$$

Numerically, this is a ~% correction, but due to the high precision of measurements, it is a very important correction.

- Oblique parameters

The EW boson self-energies encode information about <u>any</u> sector that couples to EW.

The vacuum polarizations control the quadratic part of the lagrangian,

$$\mathcal{L} \supset -\frac{1}{2} W^3_\mu \Pi_{33}(p^2) W^3_\mu - \frac{1}{2} B_\mu \Pi_{BB}(p^2) B_\mu$$

$$- W^3_\mu \Pi_{30}(p^2) B_\mu - W^+_\mu \Pi_{WW}(q^2) W^-_\mu .$$

These can be expanded as

$$\Pi(p^2) \simeq \Pi(0) + p^2 \Pi'(0) + \frac{1}{2} p^4 \Pi''(0) + \ldots$$

- Masslessnes of the photon implies two conditions on $\Pi(0)$, $\Pi_{\gamma\gamma}(0) = \Pi_{Z\gamma}(0) = 0$,

which is equivalent to

$$\frac{\Pi_{33}(0)}{c_W^2} = \frac{\Pi_{00}(0)}{s_W^2} = -\frac{\Pi_{30}(0)}{s_W c_W} \equiv \Pi_{ZZ}(0)$$

determined in terms of v.

This leaves with a single prediction given by

$$\hat{T} = \frac{\Pi_{33}(0) - \Pi_{WW}(0)}{m_W^2}$$

- $\Pi'_{WW}(0)$ and $\Pi'_{BB}(0)$ are the normalizations of the g, g' couplings. We are thus left with two predictions

$$\hat{S} = \frac{g}{g'}\,\Pi'_{30}(0) \quad , \quad \hat{U} = \Pi'_{33}(0) - \Pi'_{WW}(0)$$

It turns out that $\hat{U}$ is not very important (it is only generated by dimension 8 operators) but $\hat{S}$ & $\hat{T}$ are.

$\hat{T}$ is in fact related to ρ, $\hat{T} = 1 - \rho$.

Both have logarithmic sensitivity to the Higgs mass,

$$\hat{S} \simeq \frac{G_F m_W^2}{12\sqrt{2}\pi^2} \log m_h^2 \ , \quad \hat{T} \simeq -\frac{3 G_F m_W^2}{4\sqrt{2}\,\pi^2}\frac{g'^2}{g^2}\log m_h^2$$

Therefore, EW precision tests at LEP were able to give a lot of information about how EW symmetry seems to be broken. The best fit was

$$m_h = 85^{+39}_{-28}\ \text{GeV} \quad , \quad m_h < 165\ \text{GeV at } 95\%\ \text{CL}$$

so it seemed that there was a scalar not much above the Z boson mass. It was found later by the LHC in 2012, with $m_h \simeq 125$ GeV.

The importance of the S&T parameters are generic probes of EWSB that probe the SM structure.

From $\hat{T}$, we know that custodial symmetry seems to be a good symmetry up to ~ 10 TeV.

From $\hat{S}$, generic models of EWSB, like scenarios where the Higgs is composite, are constrained to lie above ~ 3 TeV.